Math 251
Exam 1
Spring 2016

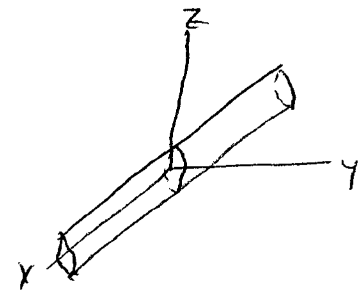
Name Key

Show All Work

1) Describe in words or by graph the region of $\mathbb{R}^3$ represented by the equation $y^2 + z^2 = 4$

circle → cylinder

A cylinder of radius 2 centered on the x-axis



2) Find the equation of a sphere if one of its diameters has endpoints (1, 2, 3) and (5, 4, -1)

center = midpoint = $\left(\frac{1+5}{2}, \frac{2+4}{2}, \frac{3-1}{2}\right) = (3, 3, 1)$

radius = $\sqrt{(3-1)^2+(3-2)^2+(1-3)^2} = \sqrt{4+1+4} = 3$

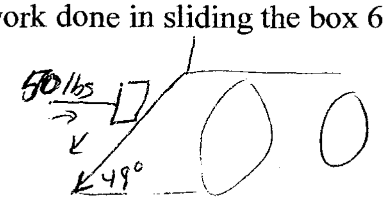
Sphere $(x-3)^2+(y-3)^2+(z-1)^2 = 9$

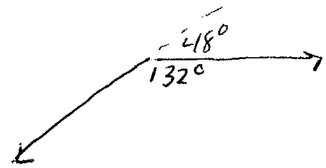
3) Find $\overrightarrow{AB}$ if A = (7, 8, 9) and B = (-1, 2, -3).

terminal − initial

$\langle -1-7, 2-8, -3-9 \rangle = \langle -8, -6, -12 \rangle$

4) A dock worker exerts a force on a box sliding down the ramp of a truck. The ramp makes an angle of 48° with the road, and the worker exerts a 50-pound force parallel to the road. Find the work done in sliding the box 6 feet.





$W = F \cdot D = |F| \cdot |D| \cdot \cos\theta$

$= 50 \cdot 6 \cdot \cos 132°$

$= -200.74$ (Since work against motion)

I accepted 200.74 using angle 48°

5) Let **a** = < 2, 2, -1 > and **b** = < 3, -4, 0 >

a) Find |**a**| and |**b**|

$|a| = \sqrt{2^2+2^2+(-1)^2} = \sqrt{9} = 3$

$|b| = \sqrt{3^2+(-4)^2+0^2} = \sqrt{25} = 5$

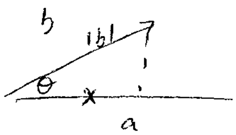
b) Find **a**·**b**

$a \cdot b = 2\cdot 3 + 2\cdot -4 + -1\cdot 0 = 6 - 8 + 0 = -2$

c) Find the angle between **a** and **b**

$\cos\theta = \dfrac{a\cdot b}{|a||b|} = \dfrac{-2}{3\cdot 5} = \dfrac{-2}{15}$ $\qquad \theta = 97.66^\circ$

d) Find the scalar projection of **b** onto **a**.



$\cos\theta = \dfrac{x}{|\vec{b}|}$

$x = |b|\cos\theta$

$x = 5\cdot\left(\dfrac{-2}{15}\right) = \underline{-\dfrac{2}{3}}$

MAY ALSO USE $\mathrm{comp}_a b = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

e) Find the vector projection of **b** onto **a**.

$(\mathrm{comp}_a b)\cdot\dfrac{a}{|a|} = \dfrac{-2}{3}\cdot\dfrac{\langle 2,2,-1\rangle}{3} = \underline{\left\langle -\dfrac{4}{9}, \dfrac{-4}{9}, \dfrac{2}{9}\right\rangle}$

ALSO $\dfrac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\,\vec{a}$

6) a) Find a nonzero vector orthogonal to the plane through the points (-1, 4, 3), (2, 0, 4), (1, 3, 1)

a b c

$\vec{ab} = \langle 3, -4, 1\rangle$

$\vec{bc} = \langle -1, 3, -3\rangle$

$$n = \begin{vmatrix} i & j & k \\ 3 & -4 & 1 \\ -1 & 3 & -3 \end{vmatrix} = \vec{i}\begin{vmatrix} -4 & 1 \\ 3 & -3 \end{vmatrix} - \vec{j}\begin{vmatrix} 3 & 1 \\ -1 & -3 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & -4 \\ -1 & 3 \end{vmatrix} = \langle 9, 8, 5\rangle$$

b) Find the linear equation of the plane going through the 3 points above.

$9(x - -1) + 8(y-4) + 5(z-3) = 0$

$9x + 9 + 8y - 32 + 5z - 15 = 0$

$\boxed{9x + 8y + 5z - 38 = 0}$

7) Find the parametric and symmetric equations of the line through (-2, 1, 3) and (1, 2, 5).

$\vec{v} = \langle 1 - -2, 2-1, 5-3 \rangle = \langle 3, 1, 2 \rangle$

parametric

$x = 3t - 2$

$y = t + 1$

$z = 2t + 3$

Symmetric

$\frac{x+2}{3} = y - 1 = \frac{z-3}{2}$

OTHER ANSWERS POSSIBLE USING VECTOR $\langle -3, -1, -2 \rangle$ AND/OR THE SECOND POINT.

8) Determine whether the following lines, L_1 and L_2, are parallel, skew, or intersecting. If they intersect, find the point of intersection.

L1: $x = 2 + 3t$, $y = -1 + 6t$, $z = 1 + 9t$

L2: $x = -3 - 2s$, $y = 1 - 4s$, $z = 1 - 6s$

$v_1 = \langle 3, 6, 9 \rangle$

$v_2 = \langle -2, -4, -6 \rangle$

$v_2 = -\frac{2}{3} v_1$

PARALLEL

9) Find the point at which the line intersects the given plane.

$x = 2 - t$, $y = 4 - 3t$, $z = 1 - 2t$ the plane $x + y + z = -5$

$2 - t + 4 - 3t + 1 - 2t = -5$

$7 - t - 3t - 2t = -5$

$-6t = -12$

$t = 2$

$x = 2 - 2 = 0$

$y = 4 - 6 = -2$

$z = 1 - 4 = -3$

POINT $(0, -2, -3)$

10) Find the angle between the planes $2x + 2y - z = 1$, $x - y + z = 1$

$\vec{n}_1 = \langle 2, 2, -1 \rangle$ $\qquad$ $n_2 = \langle 1, -1, 1 \rangle$

$|n_1| = 3$ $\qquad$ $|n_2| = \sqrt{3}$

$$\cos\theta = \frac{n_1 \cdot n_2}{|n_1||n_2|} = \frac{2-2-1}{3\sqrt{3}} = \frac{-1}{3\sqrt{3}}$$

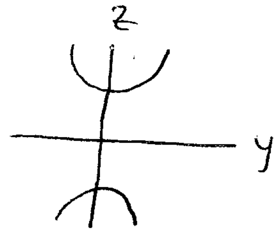
$\theta = 101.1^\circ$

OR

$\theta = 78.9^\circ$ (The acute angle)

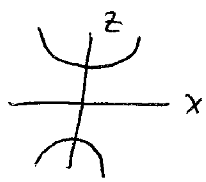
11) Graph $\frac{z^2}{4} - \frac{x^2}{9} - \frac{y^2}{4} = 1$

if $x = 0$ $\quad \frac{z^2}{4} - \frac{y^2}{4} = 1$



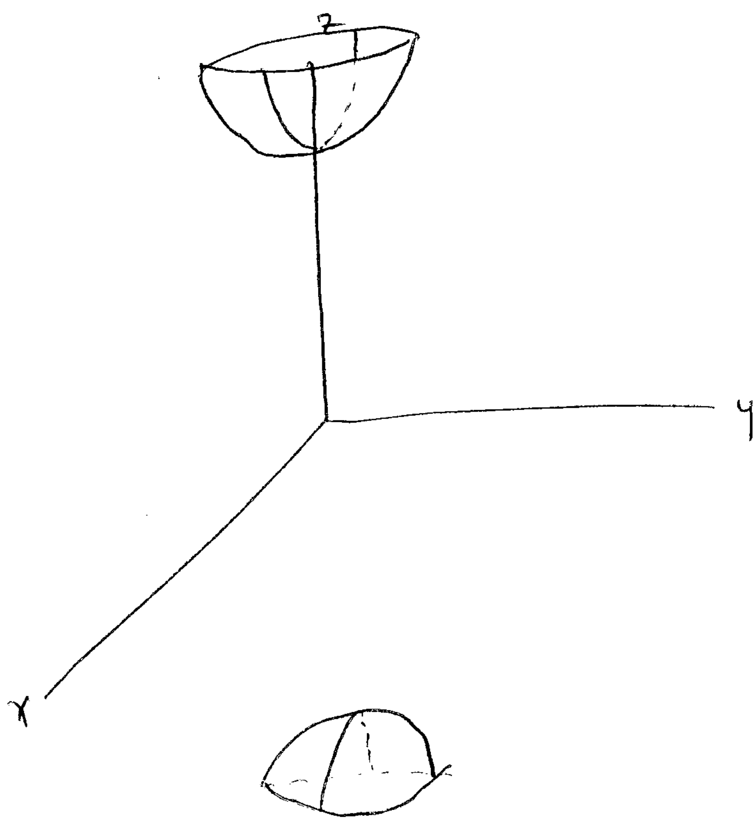
hyperbola

if $y = 0$ $\quad \frac{z^2}{4} - \frac{x^2}{9} = 1$



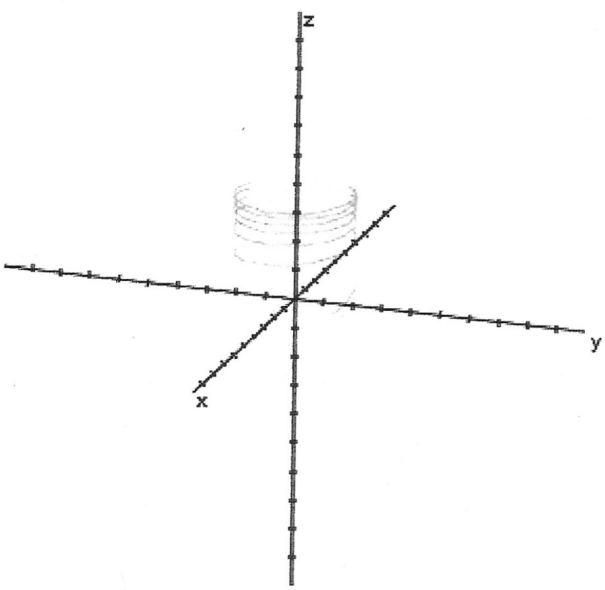
hyperbola

if $z = 0$ $\quad -\frac{x^2}{9} - \frac{y^2}{4} = 1$ $\qquad$ nothing



12) Two of the following are graphed (and two are not). Label the graphs a, b, c, or d.

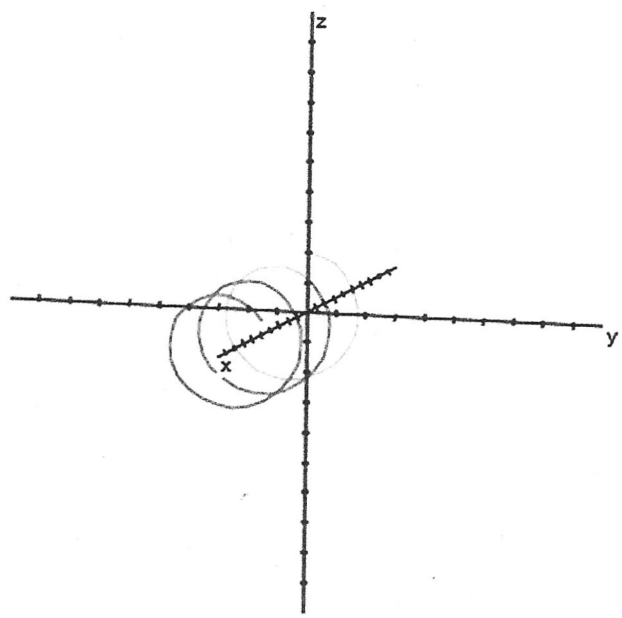
a) $x = 2\cos(2t)$, $y = 2\sin(2t)$, $z = t$ for $t > 0$
b) $x = 2\cos(2t)$, $y = 2\sin(2t)$, $z = \ln(2t)$ for $t > 0$
c) $x = t$, $y = t\cos(2t)$, $z = t\sin(2t)$ for $t > 0$
d) $x = t$, $y = 2\sin(2t)$, $z = 2\cos(2t)$ for $t > 0$



Both a and b form circles about z Axis since $x^2+y^2=4$

a is uniform
b is not

b



Both b and d form circles about x axis

in c the circles grow in radius $y^2+z^2=t^2$

in d they are uniform

d